> # Maple sample file (math mode) for J. ISCIE by M. Kanno 2011.03

> # システム/制御/情報 2011年5月号 (Vol. 55 No. 5)

> # 数式処理を用いたパラメータを有する線形制御系の解析 管野 政明

> # Section 2 - 基本操作と伝達関数

> restart :

> ## 基本操作

> $f := x^{10} - 1;$

$$f := x^{10} - 1 \quad (1)$$

> $g := factor(f);$

$$g := (x-1)\,(x+1)\,(x^4+x^3+x^2+x+1)\,(x^4-x^3+x^2-x+1) \quad (2)$$

> $h := (x+1)^2 - (x^2-1);\ simplify(h);$

$$h := (x+1)^2 - x^2 + 1$$

$$2\,x + 2 \quad (3)$$

> $g;\ expand(g);$

$$(x-1)\,(x+1)\,(x^4+x^3+x^2+x+1)\,(x^4-x^3+x^2-x+1)$$

$$x^{10} - 1 \quad (4)$$

> $\dfrac{x^2-1}{x-1};\ normal(\%);$

$$\frac{x^2-1}{x-1}$$

$$x + 1 \quad (5)$$

> ## 伝達関数

> $P := \dfrac{2}{s+2};\ K := \dfrac{1}{s};$

$$P := \frac{2}{s+2}$$

$$K := \frac{1}{s} \quad (6)$$

> $Tcl := \dfrac{P \cdot K}{1 + P \cdot K};\ Tcl := simplify(Tcl);$

$$Tcl := \frac{2}{(s+2)\,s\left(1 + \frac{2}{(s+2)\,s}\right)}$$

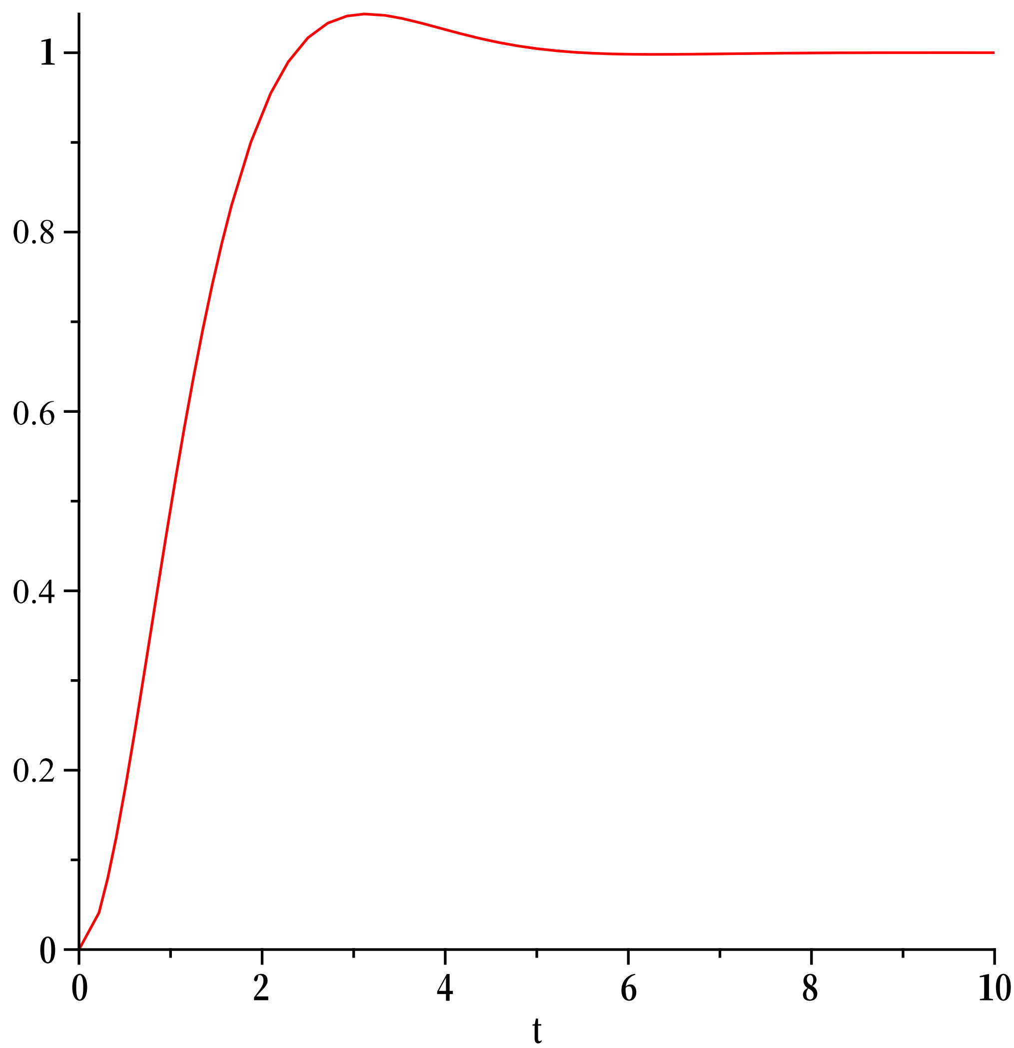
$$Tcl := \frac{2}{s^2 + 2\,s + 2} \quad (7)$$

> $with(inttrans) :$

> $Tcl\_t := invlaplace\left(Tcl \cdot \dfrac{1}{s}, s, t\right);$

$$Tcl\_t := 1 - \mathrm{e}^{-t}\,(\cos(t) + \sin(t)) \quad (8)$$

> $plot(Tcl\_t, t = 0\,..10);$



> $\lim_{t\to\infty} Tcl\_t;$

$$1 \tag{9}$$

> $\frac{d}{dt} Tcl\_t; Tcl2\_t := simplify(\%);$

$$e^{-t}(\cos(t) + \sin(t)) - e^{-t}(-\sin(t) + \cos(t))$$
$$Tcl2\_t := 2\, e^{-t} \sin(t) \tag{10}$$

> $\int_0^\infty Tcl2\_t^2 \, dt;$

$$\frac{1}{2} \tag{11}$$

> $g\_t := invlaplace\left(\frac{1}{s+a}, s, t\right);$

$$g\_t := e^{-a\,t} \tag{12}$$

> $\lim_{t\to\infty} g\_t;$

$$\lim_{t\to\infty} e^{-a\,t} \tag{13}$$

> $\lim_{t\to\infty} g\_t$ assuming $a > 0;$

$$0 \tag{14}$$

> $invlaplace\left(\frac{1}{s^3+2\cdot s^2+3\cdot s+4}, s, t\right);$

$$\frac{1}{20}\sum_{\_\alpha=RootOf(\_Z^3+2\_Z^2+3\_Z+4)} e^{\_\alpha t}\left(-\_\alpha+\_\alpha^2\right) \quad (15)$$

> $evalf(\%);$

$$0.2187602960\, e^{-1.650629191\, t} + (-0.1093801480 + 0.1043649860\, I)\, e^{(-0.1746854043 - 1.546868887\, I)\, t} + (-0.1093801480 - 0.1043649860\, I)\, e^{(-0.1746854043 + 1.546868887\, I)\, t} \quad (16)$$

> # *Section 3 - パラメータを有する線形系の安定性*

> $with(PolynomialTools) : with(LinearAlgebra) :$

> **read** "HurwitzMatrix.maple";

> $[seq(a \| i, i=0..5)]; p := sort(FromCoefficientList(\%, s));$

$$[a0, a1, a2, a3, a4, a5]$$

$$p := a5\, s^5 + a4\, s^4 + a3\, s^3 + a2\, s^2 + a1\, s + a0 \quad (17)$$

> $Hn := HurwitzMatrix(p, s);$

$$Hn := \begin{bmatrix} a4 & a2 & a0 & 0 & 0 \\ a5 & a3 & a1 & 0 & 0 \\ 0 & a4 & a2 & a0 & 0 \\ 0 & a5 & a3 & a1 & 0 \\ 0 & 0 & a4 & a2 & a0 \end{bmatrix} \quad (18)$$

> # *if 文を用いたプロシージャ*

> **read** "HurwitzMatrix2.maple";

> $Hn2 := HurwitzMatrix2(p, s);$

$$Hn2 := \begin{bmatrix} a4 & a2 & a0 & 0 & 0 \\ a5 & a3 & a1 & 0 & 0 \\ 0 & a4 & a2 & a0 & 0 \\ 0 & a5 & a3 & a1 & 0 \\ 0 & 0 & a4 & a2 & a0 \end{bmatrix} \quad (19)$$

> $Hn - Hn2;$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (20)$$

> # *同じ結果が得られた*

> $p := convert\left(s^3 + (1.03 + q)\cdot s^2 + (1.02 + 3\cdot q)\cdot s + (1.01 + 7\cdot q), rational, exact\right);$

$$p := s^3 + \left(\frac{103}{100} + q\right) s^2 + \left(\frac{51}{50} + 3\, q\right) s + \frac{101}{100} + 7\, q \quad (21)$$

> $Hn := HurwitzMatrix(p, s);$

$$Hn := \begin{bmatrix} \frac{103}{100}+q & \frac{101}{100}+7q & 0 \\ 1 & \frac{51}{50}+3q & 0 \\ 0 & \frac{103}{100}+q & \frac{101}{100}+7q \end{bmatrix} \tag{22}$$

> $f := sort(Determinant(Hn));$

$$f := 21q^3 - \frac{86}{5}q^2 - \frac{26347}{10000}q + \frac{20503}{500000} \tag{23}$$

> $roots\_f := sort(realroot(f, 10^{-8})); evalf(roots\_f);$

$$roots\_f := \left[\left[-\frac{19365701}{134217728}, -\frac{4841425}{33554432}\right], \left[\frac{1913879}{134217728}, \frac{239235}{16777216}\right], \left[\frac{127382531}{134217728}, \frac{31845633}{33554432}\right]\right]$$

$$[[-0.1442857161, -0.1442857087], [0.01425950974, 0.01425951719], [0.9490738139, 0.9490738213]] \tag{24}$$

> $subs(q = roots\_f[1][1] - 1, p); Hurwitz(\%, s);$

$$s^3 - \frac{383479229}{3355443200}s^2 - \frac{8096205111}{3355443200}s - \frac{23488102443}{3355443200}$$

$$\text{false} \tag{25}$$

> $Hurwitz\left(subs\left(q = \frac{roots\_f[1][2] + roots\_f[2][1]}{2}, p\right), s\right);$

$$\text{true} \tag{26}$$

> $Hurwitz\left(subs\left(q = \frac{roots\_f[2][2] + roots\_f[3][1]}{2}, p\right), s\right);$

$$\text{false} \tag{27}$$

> $Hurwitz(subs(q = roots\_f[3][2] + 1, p), s);$

$$\text{true} \tag{28}$$

> # Section 4 - パラメータを有する線形系のH2ノルム

> $A := \begin{bmatrix} -a & -1 \\ 1 & 0 \end{bmatrix}; B := \begin{bmatrix} 1 \\ 0 \end{bmatrix}; C := \begin{bmatrix} 2 & -1 \end{bmatrix};$

$$A := \begin{bmatrix} -a & -1 \\ 1 & 0 \end{bmatrix}$$

$$B := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 2 & -1 \end{bmatrix} \tag{29}$$

> $sort(simplify(C.(s \cdot IdentityMatrix(2) - A)^{-1}.B), s);$

$$\frac{2s-1}{s^2 + a s + 1} \tag{30}$$

> $Lo := Matrix(2, symbol = l, shape = symmetric);$

(31)

$$Lo := \begin{bmatrix} l_{1,1} & l_{1,2} \\ l_{1,2} & l_{2,2} \end{bmatrix} \tag{31}$$

> ## *solve* の使い方

> $solve(x^2-2);$

$$\sqrt{2},\ -\sqrt{2} \tag{32}$$

> $solve(x^2-a, x);$

$$\sqrt{a},\ -\sqrt{a} \tag{33}$$

> $solve(x^2-a);$

$$\{a = x^2, x = x\} \tag{34}$$

> $solve(x^2-a, \{x\});$

$$\{x = \sqrt{a}\},\ \{x = -\sqrt{a}\} \tag{35}$$

> $solve(\{2 \cdot x + y = 3, x - y = 0\});$

$$\{x = 1, y = 1\} \tag{36}$$

> ## *solve* の使い方 - 終わり

> $Lyap := Transpose(A).Lo + Lo.A + Transpose(C).C;$

$$Lyap := \begin{bmatrix} -2\,a\,l_{1,1} + 2\,l_{1,2} + 4 & -a\,l_{1,2} + l_{2,2} - l_{1,1} - 2 \\ -a\,l_{1,2} + l_{2,2} - l_{1,1} - 2 & -2\,l_{1,2} + 1 \end{bmatrix} \tag{37}$$

> $convert(Lo, set);$

$$\{l_{1,1}, l_{1,2}, l_{2,2}\} \tag{38}$$

> $sol := solve(convert(Lyap, set), convert(Lo, set));$

$$sol := \left\{l_{1,1} = \frac{5}{2\,a}, l_{1,2} = \frac{1}{2}, l_{2,2} = \frac{1}{2}\,\frac{a^2+5+4\,a}{a}\right\} \tag{39}$$

> $subs(sol, Lo); Transpose(B).\%.B;$

$$\begin{bmatrix} \frac{5}{2\,a} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2}\,\frac{a^2+5+4\,a}{a} \end{bmatrix}$$

$$\frac{5}{2\,a} \tag{40}$$

> # end

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