```
> # Maple sample file (text mode) for J. ISCIE by M. Kanno 2011.03
> # システム/制御/情報 2011年5月号 (Vol. 55 No. 5)
> # 数式処理を用いたパラメータを有する線形制御系の解析　管野 政明
> # Section 2 - 基本操作と伝達関数
> restart;
> ## 基本操作
> f := x^10 - 1;
```

$$f := x^{10} - 1 \tag{1}$$

```
> g := factor(f);
```

$$g := (x-1)\,(x+1)\,\left(x^4+x^3+x^2+x+1\right)\,\left(x^4-x^3+x^2-x+1\right) \tag{2}$$

```
>  h := (x+1)^2 - x^2 + 1; simplify(h);
```

$$h := (x+1)^2 - x^2 + 1$$

$$2\,x+2 \tag{3}$$

```
> g; expand(g);
```

$$(x-1)\,(x+1)\,\left(x^4+x^3+x^2+x+1\right)\,\left(x^4-x^3+x^2-x+1\right)$$

$$x^{10}-1 \tag{4}$$

```
> (x^2 - 1) / (x - 1); normal(%);
```

$$\frac{x^2-1}{x-1}$$

$$x+1 \tag{5}$$

```
> ## 伝達関数
> P := 2 / (s + 2); K := 1 / s;
```

$$P := \frac{2}{s+2}$$

$$K := \frac{1}{s} \tag{6}$$

```
> Tcl := P*K / (1 + P*K); Tcl := simplify(Tcl);
```

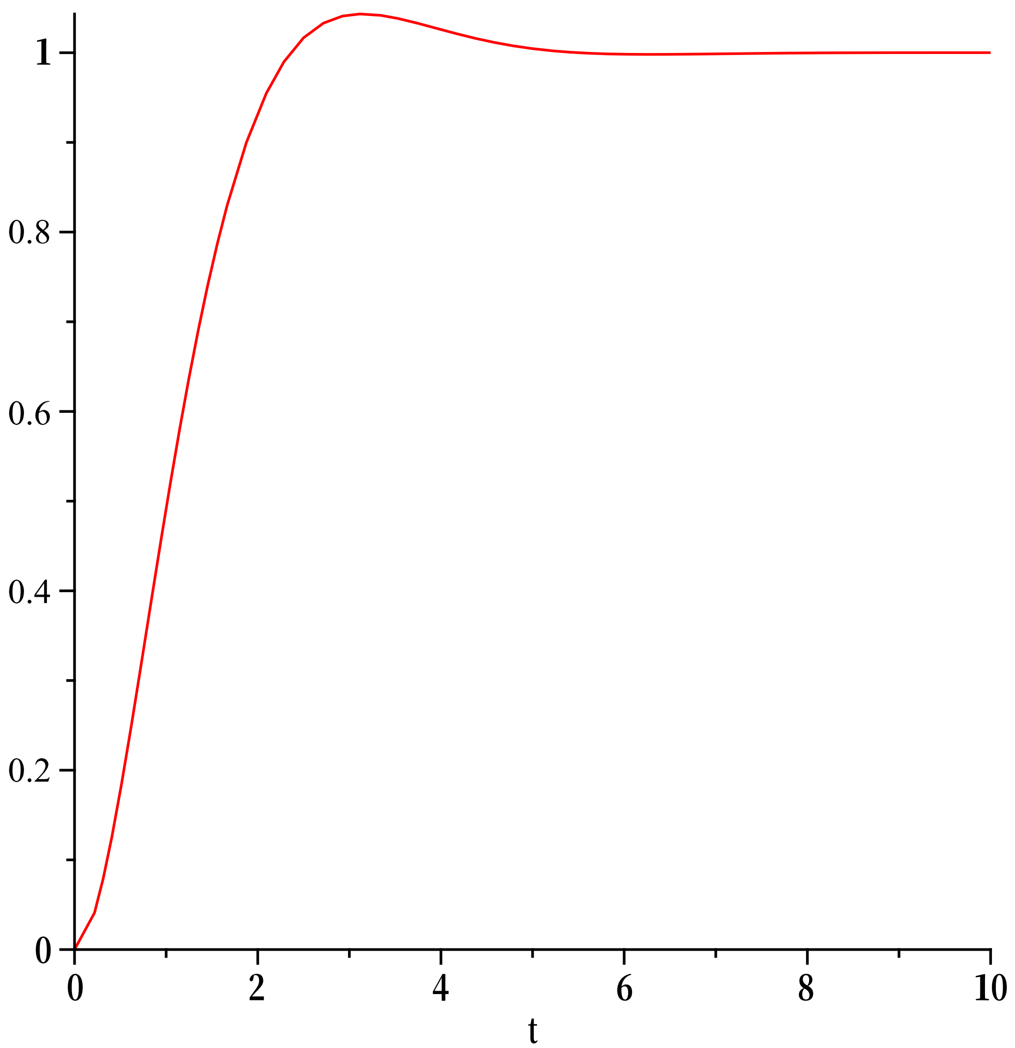
$$Tcl := \frac{2}{(s+2)\,s\,\left(1+\frac{2}{(s+2)\,s}\right)}$$

$$Tcl := \frac{2}{s^2+2\,s+2} \tag{7}$$

```
> with(inttrans):
> Tcl_t := invlaplace(Tcl*1/s, s, t);
```

$$Tcl\_t := 1 - \mathrm{e}^{-t}\,(\cos(t) + \sin(t)) \tag{8}$$

```
> plot(Tcl_t, t = 0..10);
```




```
> limit(Tcl_t, t = infinity);
```

$$1 \tag{9}$$

```
> diff(Tcl_t, t); Tcl2_t := simplify(%);
```

$$e^{-t}\left(\cos(t)+\sin(t)\right)-e^{-t}\left(-\sin(t)+\cos(t)\right)$$

$$Tcl2\_t := 2\,e^{-t}\sin(t) \tag{10}$$

```
> int(Tcl2_t^2, t = 0..infinity);
```

$$\frac{1}{2} \tag{11}$$

```
> g_t := invlaplace(1 / (s + a), s, t);
```

$$g\_t := e^{-a\,t} \tag{12}$$

```
> limit(g_t, t = infinity);
```

$$\lim_{t\to\infty} e^{-a\,t} \tag{13}$$

```
> limit(g_t, t = infinity) assuming a > 0;
```

$$0 \tag{14}$$

```
> invlaplace(1 / (s^3 + 2*s^2 + 3*s + 4), s, t);
```

$$\frac{1}{20}\sum_{\_\alpha = RootOf\left(\_Z^3+2\,\_Z^2+3\,\_Z+4\right)} e^{\_\alpha\,t}\left(-\_\alpha+\_\alpha^2\right) \tag{15}$$

```
> evalf(%);
```

$$0.2187602960\,e^{-1.650629191\,t}+(-0.1093801480 \tag{16}$$
$$+0.1043649860\,\mathrm{I})\,e^{(-0.1746854043-1.546868887\,\mathrm{I})\,t}+(-0.1093801480$$

$$- 0.1043649860\,I)\, e^{(-0.1746854043 + 1.546868887\,I)\,t}$$

```
> # Section 3 - パラメータを有する線形系の安定性
> with(PolynomialTools): with(LinearAlgebra):
> read "HurwitzMatrix.maple";
> [seq(a||i, i = 0..5)]; p := sort(FromCoefficientList(%, s));
```

$$[a0, a1, a2, a3, a4, a5]$$

$$p := a5\, s^5 + a4\, s^4 + a3\, s^3 + a2\, s^2 + a1\, s + a0 \tag{17}$$

```
> Hn := HurwitzMatrix(p, s);
```

$$Hn := \begin{bmatrix} a4 & a2 & a0 & 0 & 0 \\ a5 & a3 & a1 & 0 & 0 \\ 0 & a4 & a2 & a0 & 0 \\ 0 & a5 & a3 & a1 & 0 \\ 0 & 0 & a4 & a2 & a0 \end{bmatrix} \tag{18}$$

```
> # if 文を用いたプロシージャ
> read "HurwitzMatrix2.maple";
> Hn2 := HurwitzMatrix2(p, s);
```

$$Hn2 := \begin{bmatrix} a4 & a2 & a0 & 0 & 0 \\ a5 & a3 & a1 & 0 & 0 \\ 0 & a4 & a2 & a0 & 0 \\ 0 & a5 & a3 & a1 & 0 \\ 0 & 0 & a4 & a2 & a0 \end{bmatrix} \tag{19}$$

```
> Hn - Hn2;
```

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{20}$$

```
> # 同じ結果が得られた
> p := convert(s^3 + (1.03 + q)*s^2 + (1.02 + 3*q)*s + (1.01 + 7*q),
  rational, exact);
```

$$p := s^3 + \left(\frac{103}{100} + q\right) s^2 + \left(\frac{51}{50} + 3\,q\right) s + \frac{101}{100} + 7\,q \tag{21}$$

```
> Hn := HurwitzMatrix(p, s);
```

$$Hn := \begin{bmatrix} \frac{103}{100} + q & \frac{101}{100} + 7\,q & 0 \\ 1 & \frac{51}{50} + 3\,q & 0 \\ 0 & \frac{103}{100} + q & \frac{101}{100} + 7\,q \end{bmatrix} \tag{22}$$

```
> f := sort(Determinant(Hn));
```

$$f := 21\, q^3 - \frac{86}{5}\, q^2 - \frac{26347}{10000}\, q + \frac{20503}{500000} \tag{23}$$

```
> roots_f := sort(realroot(f, 10^(-8))); evalf(roots_f);
```

$$roots\_f := \left[\left[-\frac{19365701}{134217728}, -\frac{4841425}{33554432}\right], \left[\frac{1913879}{134217728}, \frac{239235}{16777216}\right], \left[\frac{127382531}{134217728}, \frac{31845633}{33554432}\right]\right]$$

$$[[-0.1442857161, -0.1442857087], [0.01425950974, 0.01425951719], [0.9490738139, 0.9490738213]] \tag{24}$$

```
> subs(q = roots_f[1][1] - 1, p); Hurwitz(%, s);
```

$$s^3 - \frac{383479229}{3355443200} s^2 - \frac{8096205111}{3355443200} s - \frac{23488102443}{3355443200}$$

$$\text{false} \tag{25}$$

```
> Hurwitz(subs(q = (roots_f[1][2] + roots_f[2][1])/2, p), s);
```

$$\text{true} \tag{26}$$

```
> Hurwitz(subs(q = (roots_f[2][2] + roots_f[3][1])/2, p), s);
```

$$\text{false} \tag{27}$$

```
> Hurwitz(subs(q = roots_f[3][2] + 1, p), s);
```

$$\text{true} \tag{28}$$

```
> # Section 4 - パラメータを有する線形系のH2ノルム
> A := Matrix([[-a, -1], [1, 0]]); B := Vector([1, 0]); C := Vector(
  [2, -1], orientation = row);
```

$$A := \begin{bmatrix} -a & -1 \\ 1 & 0 \end{bmatrix}$$

$$B := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C := \begin{bmatrix} 2 & -1 \end{bmatrix} \tag{29}$$

```
> sort(simplify(C.(s*IdentityMatrix(2) - A)^(-1).B), s);
```

$$\frac{2 s - 1}{s^2 + a s + 1} \tag{30}$$

```
> Lo := Matrix(2, symbol = l, shape = symmetric);
```

$$Lo := \begin{bmatrix} l_{1,1} & l_{1,2} \\ l_{1,2} & l_{2,2} \end{bmatrix} \tag{31}$$

```
> ## solve の使い方
> solve(x^2 - 2);
```

$$\sqrt{2}, -\sqrt{2} \tag{32}$$

```
> solve(x^2 - a, x);
```

$$\sqrt{a}, -\sqrt{a} \tag{33}$$

```
> solve(x^2 - a);
```

$$\{a = x^2, x = x\} \tag{34}$$

```
> solve(x^2 - a, {x});
```

$$\{x = \sqrt{a}\}, \{x = -\sqrt{a}\} \tag{35}$$

```
> solve({x - y = 0, 2*x + y = 3});
```

$$\{x = 1, y = 1\} \tag{36}$$

```
> ## solve の使い方 - 終わり
> Lyap := Transpose(A).Lo + Lo.A + Transpose(C).C;
```

$$Lyap := \begin{bmatrix} -2\,a\,l_{1,1}+2\,l_{1,2}+4 & -a\,l_{1,2}+l_{2,2}-l_{1,1}-2 \\ -a\,l_{1,2}+l_{2,2}-l_{1,1}-2 & -2\,l_{1,2}+1 \end{bmatrix} \tag{37}$$

```
> convert(Lo, set);
```

$$\{l_{1,1}, l_{1,2}, l_{2,2}\} \tag{38}$$

```
> sol := solve(convert(Lyap, set), convert(Lo, set));
```

$$sol := \left\{l_{1,1} = \frac{5}{2\,a}, l_{1,2} = \frac{1}{2}, l_{2,2} = \frac{1}{2}\,\frac{a^2+5+4\,a}{a}\right\} \tag{39}$$

```
> subs(sol, Lo); Transpose(B).%.B;
```

$$\begin{bmatrix} \frac{5}{2\,a} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2}\,\frac{a^2+5+4\,a}{a} \end{bmatrix}$$

$$\frac{5}{2\,a} \tag{40}$$

```
> # end
```